

Grade 4 Mathematics Curriculum Guide

King George County Schools

2011

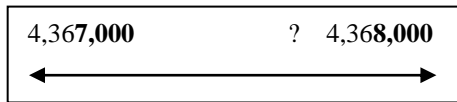
<p>Standard</p>	<p>MATH 4.1</p> <p>The student will</p> <ul style="list-style-type: none"> a) identify orally and in writing the place value for each digit in a whole number expressed through millions. b) compare two whole numbers expressed through millions, using symbols ($>$, $<$, or $=$). c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand. <p>Timeframe: Q1 (5 days)</p>			<p>Essential Understandings</p> <p>All students should</p> <ul style="list-style-type: none"> • Understand the relationships in the place value system in which the value of each place is ten times the value of the place to its right. • Use the patterns in the place value system to read and write numbers. • Understand that reading place value correctly is essential when comparing numbers. • Understand that rounding gives a close number to use when exact numbers are not needed for the situation at hand. • Develop strategies for rounding. <p>Teacher Notes</p> <p>See next page for notes.</p>					
<p>Strand</p>	<p>Number and Number Sense</p>								
<p>Essential Knowledge and Skills</p>	<ul style="list-style-type: none"> a) Identify and communicate, both orally and in written form, the placed value for each digit in whole numbers expressed through the one millions place. b) Read whole numbers through the one millions place that are presented in standard format, and select the matching number in written format. c) Write whole numbers through the one millions place in standard format when the numbers are presented orally or in written format. d) Identify and use the symbols for <i>greater than</i>, <i>less than</i>, and <i>equal to</i>. e) Compare two whole numbers expressed through the one millions, using symbols $>$, $<$, or $=$. f) Round whole numbers expressed through the one millions place to the nearest thousand, ten thousand, and hundred-thousand place. 								
<p>Resources</p>	<table border="1"> <thead> <tr> <th data-bbox="325 1234 854 1317">Resource Materials</th> <th data-bbox="860 1234 1252 1317">Released Test Items</th> <th data-bbox="1257 1234 1704 1317">Individual Teacher Notes</th> </tr> </thead> <tbody> <tr> <td data-bbox="325 1320 854 1479"> <p><u>Envision Textbook</u> 4.1a</p> <ul style="list-style-type: none"> • SE: 4-9, 20-21 </td> <td data-bbox="860 1320 1252 1479"> <p>2010: 14, 15, 16 2009: 13, 17, 19</p> </td> <td data-bbox="1257 1320 1704 1479"></td> </tr> </tbody> </table>	Resource Materials	Released Test Items	Individual Teacher Notes	<p><u>Envision Textbook</u> 4.1a</p> <ul style="list-style-type: none"> • SE: 4-9, 20-21 	<p>2010: 14, 15, 16 2009: 13, 17, 19</p>			
Resource Materials	Released Test Items	Individual Teacher Notes							
<p><u>Envision Textbook</u> 4.1a</p> <ul style="list-style-type: none"> • SE: 4-9, 20-21 	<p>2010: 14, 15, 16 2009: 13, 17, 19</p>								

	<ul style="list-style-type: none"> • SEVH: VA 14 <p>4.1b</p> <ul style="list-style-type: none"> • SE: 10-13, 24 (Set B), 39, 373 • SEVH: VA 15 <p>4.1c</p> <ul style="list-style-type: none"> • SE: 14-15, 32-33, 39, 100-103, 144-145, 166-167, 294-295 • SEVH: VA 16 • http://ttaonline.org/ 	<p>2008: 13, 17</p> <p>2007: 13. 16</p>		
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Teacher Notes

- The structure of the Base-10 number system is based upon a simple pattern of tens, in which the value of each place is ten times the value of the place to its right.
- Place value refers to the value of each digit and depends upon the position of the digit in the number. For example, in the number 7,864,352, the eight is in the hundred thousands place, and the value of the 8 is eight hundred thousand or 800,000.
- Whole numbers may be written in a variety of formats:
 - Standard: 1,234,567
 - Written: one million, two hundred thirty-four thousand, five hundred sixty-seven
 - Expanded: $(1 \times 1,000,000) + (2 \times 100,000) + (3 \times 10,000) + (4 \times 1,000) + (5 \times 100) + (6 \times 10) + (7 \times 1)$
- Numbers are arranged into groups of three places called *periods* (ones, thousands, millions, ...). Places within the periods repeat (hundreds, tens, ones). Commas are used to separate the periods. Knowing the place value and period of a number helps students find values of digits in any number as well as read and write numbers.
- Reading and writing large numbers should be meaningful for students. Experiences can be provided that relate practical situations (e.g., numbers found in the students' environment including population, number of school lunches sold statewide in a day, etc.). Concrete materials such as Base-10 blocks and bundles of sticks may be used to represent whole numbers through thousands. Larger numbers may be represented by digit cards and place value charts.

- Mathematical symbols ($>$, $<$) used to compare two unequal numbers are called *inequality symbols*.
- A procedure for comparing two numbers by examining place value may include the following:
 - Compare the digits in the numbers to determine which number is greater (or which is less).
 - Use a number line to identify the appropriate placement of the numbers based on the place value of the digits.
 - Use the appropriate symbol $>$ or $<$ or words *greater than* or *less than* to compare the numbers in the order in which they are presented.
 - If both numbers have the same value, use the symbol $=$ or words *equal to*.
- A strategy for rounding numbers to the nearest thousand, ten thousand, and hundred thousand is as follows:
 - Use a number line to determine the rounded number (e.g., when rounding 4,367,925 to the nearest thousand, identify the ‘thousands’ the number would fall between on the number line, then determine the thousand that the number is closest to):



- Look one place to the right of the digit to which you wish to round.
- If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero.
- If the digit is 5 or greater, add 1 to the digit in the rounding place and change the digits to the right of the rounding place to zero.

Essential New Vocabulary

- | | |
|---|---|
| <ul style="list-style-type: none"> • period (periods) • whole number (whole numbers) • greater than; less than • equal to • million (millions) • place value • round | <ul style="list-style-type: none"> • symbols ($<$, $>$, $=$) • thousand • ten thousand • hundred thousand • standard form • written form • base ten |
|---|---|

Released Test Items

A city has a population of 7,380,916. What is 7,380,916 rounded to the nearest hundred thousand?

- A** 7,000,000
- B** 7,380,900
- C** 7,400,000
- D** 8,000,000

<p>Standard</p>	<p>MATH 4.2</p> <p>The student will</p> <p>a) compare and order fractions and mixed numbers. b) represent equivalent fractions. c) identify the division statement that represents a fraction.</p> <p>Timeframe: Q 3 (6 days)</p>	<p>Essential Understandings</p> <p>All students should</p> <ul style="list-style-type: none"> • Develop an understanding of fractions as parts of unit wholes, as parts of a collection, and as locations on a number line. • Understand that a mixed number is a fraction that has two parts: a whole number and a proper fraction. The mixed number is the sum of these two parts. • Use models, benchmarks, and equivalent forms to judge the size of fractions. • Recognize that a whole divided into nine equal parts has smaller parts than if the whole had been divided into five equal parts. • Recognize and generate equivalent forms of commonly used fractions and decimals. • Understand the division statement that represents a fraction. • Understand that the more parts the whole is divided into, the smaller the parts (e.g., $\frac{1}{5} < \frac{1}{3}$).
<p>Strand</p>	<p>Number and Number Sense</p>	
<p>Essential Knowledge and Skills</p>	<p>a) Compare and order fractions having denominators of 12 or less, using manipulative models and drawings, such as region/area models.</p> <p>b) Compare and order fractions with like denominators by comparing number of parts (numerators) (e.g., $\frac{1}{5} < \frac{3}{5}$).</p> <p>c) Compare and order fractions with like numerators and unlike denominators by comparing the size of the parts (e.g., $\frac{3}{9} < \frac{3}{5}$).</p> <p>d) Compare and order fractions having unlike denominators of 12 or less by comparing the fractions to benchmarks</p> <p>e) (e.g., 0, $\frac{1}{2}$ or 1) to determine their relationships to the benchmarks or by finding a common denominator.</p> <p>f) Compare and order mixed numbers having denominators of 12 or less.</p> <p>g) Use the symbols $>$, $<$, and $=$ to compare the numerical value of fractions and mixed numbers having denominators of 12 or less.</p> <p>h) Represent equivalent fractions through twelfths, using region/area models, set models, and measurement models.</p> <p>i) Identify the division statement that represents a fraction (e.g., $\frac{3}{5}$ means the same as 3 divided by 5).</p>	<p>Teacher Notes</p> <p>See next page for notes.</p>

	Resource Materials	Released Test Items	Individual Teacher Notes
Resources	<p><u>Envision Textbook</u></p> <p>4.2a</p> <ul style="list-style-type: none"> SE: 234-235, 236-237, 246 (Set G), 247 (Set H), 276-278, 280-281, 282-283 SEVH: VA 29 <p>4.2b</p> <ul style="list-style-type: none"> SE: 224-227, 238, 241, 245 (Set D) SEVH: VA 28 <p>4.2c</p> <ul style="list-style-type: none"> SE: 216-218, 220, 221, 244 (Set A) SEVH: VA 10-VA 11, VA 27 http://ttaonline.org/ 	<p>2010: 13, 17, 20</p> <p>2009: 15, 18</p> <p>2008:</p> <p>2007: 17, 18</p>	

Teacher Notes

- A fraction is a way of representing part of a whole (as in a region/area model or a measurement model) or part of a group (as in a set model). A fraction is used to name a part of one thing or a part of a collection of things.
- In the area/region and length/measurement fraction models, the parts must be equal. In the set model, the elements of the set do not have to be equal (i.e., "What fraction of the class is wearing the color red?").
- The denominator tells how many equal parts are in the whole or set. The numerator tells how many of those parts are being counted or described.
- When fractions have the same denominator, they are said to have "common denominators" or "like denominators." Comparing fractions with like denominators involves comparing only the numerators.

- Strategies for comparing fractions having unlike denominators may include:
 - Comparing fractions to familiar benchmarks (e.g., 0, $\frac{1}{2}$, 1)
 - Finding equivalent fractions, using manipulative models such as fraction strips, number lines, fraction circles, rods, pattern blocks, cubes, Base-10 blocks, tangrams, graph paper, or a multiplication chart and patterns
 - Finding a common denominator by finding the least common multiple (LCM) of both denominators and then rewriting each fraction as an equivalent fraction, using the LCM as the denominator.
- A variety of fraction models should be used to expand students' understanding of fractions and mixed numbers:
 - Region/area models: a surface or area is subdivided into smaller equal parts, and each part is compared with the whole (e.g., fraction circles, pattern blocks, geoboards, grid paper, color tiles)
 - Set models: the whole is understood to be a set of objects, and subsets of the whole make up fractional parts (e.g., counters, chips)
 - Measurement models: similar to area models but lengths instead of areas are compared (e.g., fraction strips, rods, cubes, number lines, rulers).
- A mixed number has two parts: a whole number and a fraction.
- Equivalent fractions name the same amount. Students should use a variety of models to identify different names for equivalent fractions.
- Students should focus on finding equivalent fractions of familiar fractions such as halves, thirds, fourths, sixths, eighths, tenths, and twelfths.
- Decimals and fractions represent the same relationships; however, they are presented in two different formats. The decimal 0.25 is written as $\frac{1}{4}$. When presented with the fraction $\frac{3}{5}$, the division expression representing a fraction is written as 3 divided by 5.

Essential New Vocabulary

- | | |
|---------------------------------------|------------------------|
| • denominator | • number line |
| • numerator | • unlike denominators |
| • common denominator(s) of 12 or less | • mixed number |
| • like numerators | • equivalent fractions |

<p>Standard</p>	<p>MATH 4.3</p> <p>The student will</p> <p>a) read, write, represent, and identify decimals expressed through thousandths.</p> <p>b) round decimals to the nearest whole number, tenth, and hundredth.</p> <p>c) compare and order decimals.</p> <p>d) given a model, write the decimal and fraction equivalents.</p> <p>Timeframe: Q 1 (5 days) Q 2 (4 days)</p>	<p>Essential Understandings</p> <p>All students should</p> <ul style="list-style-type: none"> • Understand the place value structure of decimals and use this structure to read, write, and compare decimals. • Understand that decimal numbers can be rounded to an estimate when exact numbers are not needed for the situation at hand. • Understand that decimals are rounded in a way that is similar to the way whole numbers are rounded. • Understand that decimals and fractions represent the same relationship; however, they are presented in two different formats. • Understand that models are used to show decimal and fraction equivalents.
<p>Strand</p>	<p>Number and Number Sense</p>	
<p>Essential Knowledge and Skills</p>	<p>a) Investigate the ten-to-one place value relationship for decimals through thousandths, using Base-10 manipulatives (e.g., place value mats/charts, decimal squares, Base-10 blocks, money).</p> <p>b) Represent and identify decimals expressed through thousandths, using Base-10 manipulatives, pictorial representations, and numerical symbols (e.g., relate the appropriate drawing to 0.05).</p> <p>c) Identify and communicate, both orally and in written form, the position and value of a decimal through thousandths. For example, in 0.385, the 8 is in the hundredths place and has a value of 0.08.</p> <p>d) Read and write decimals expressed through thousandths, using Base-10 manipulatives, drawings, and numerical symbols.</p> <p>e) Round decimals to the nearest whole number, tenth, and hundredth.</p> <p>f) Compare decimals, using the symbols $>$, $<$, $=$.</p> <p>g) Order a set of decimals from least to greatest or greatest to least.</p> <p>h) Represent fractions for halves, fourths, fifths, and tenths as decimals through hundredths, using concrete objects (e.g., demonstrate the relationship between the fraction $\frac{1}{4}$ and its decimal equivalent 0.25).</p>	<p>Teacher Notes</p> <p>See next page for notes.</p>

	<p>i) Relate fractions to decimals, using concrete objects (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, decimal circles, money [coins]).</p> <p>j) Write the decimal and fraction equivalent for a given model (e.g., $\frac{1}{4} = 0.25$ or $0.25 = \frac{1}{4}$).</p>			
Resources	Resource Materials	Released Test Items	Individual Teacher Notes	
	<p><u>Envision Textbook</u></p> <p>4.3a</p> <ul style="list-style-type: none"> • SE: 16, 17, 25 (set D), 268, 269, 274-275, 286 (Sets A and C) • SEVH VA 31 <p>4.3b</p> <ul style="list-style-type: none"> • SE: 290-292, 294, 295, 300-301 • SEVH: VA 36-37, VA 49 <p>4.3c</p> <ul style="list-style-type: none"> • SE: 270-271, 272, 276-277, 278, 280-281, 282-283, 286 (set B) • SEVH: VA 32 <p>4.3d</p> <ul style="list-style-type: none"> • SE: SE: 274-278, 280, 281 • SEVH: VA 33 • http://ttaconline.org/ 	<p>2010: 18, 19</p> <p>2009: 16, 20</p> <p>2008:14, 15, 19, 20</p> <p>2007: 14, 19</p>		

Teacher Notes

- The structure of the Base-10 number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This is known as a ten-to-one place value relationship.
- Understanding the system of tens means that ten tenths represents one whole, ten hundredths represents one tenth, ten thousandths represents one hundredth.
- A decimal point separates the whole number places from the places that are less than one. Place values extend infinitely in two directions from a decimal point. A number containing a decimal point is called a *decimal number* or simply a *decimal*.
- To read decimals,
 - read the whole number to the left of the decimal point, if there is one.
 - read the decimal point as “and”.
 - read the digits to the right of the decimal point just as you would read a whole number.
 - say the name of the place value of the digit in the smallest place.
- Any decimal less than 1 will include a leading zero (e.g., 0.125).
- Decimals may be written in a variety of forms:
 - Standard: 26.537
 - Written: twenty-six and five hundred thirty-seven thousandths
 - Expanded: $(2 \times 10) + (6 \times 1) + (5 \times 0.1) + (3 \times 0.01) + (7 \times 0.001)$.
- Decimals and fractions represent the same relationships; however, they are presented in two different formats. The decimal 0.25 is written as $\frac{1}{4}$. Decimal numbers are another way of writing fractions. When presented with the fraction $\frac{3}{5}$, the division expression representing a fraction is written as 3 divided by 5. The Base-10 models concretely relate fractions to decimals (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money).
- The procedure for rounding decimal numbers is similar to the procedure for rounding whole numbers.
- A strategy for rounding decimal numbers to the nearest tenth and hundredth is as follows:
 - Look one place to the right of the digit you want to round to.
 - If the digit is 5 or greater, add 1 to the digit in the rounding place, and drop the digits to the right of the rounding place.
 - If the digit is less than 5, leave the digit in the rounding place as it is, and drop the digits to the right of the rounding place.
- Different strategies for rounding decimals include:
 - Use a number line to locate a decimal between two numbers. For example, 18.83 is closer to 18.8 than to 18.9.
 - Compare the digits in the numbers to determine which number is greater (or which is less).
 - Compare the value of decimals, using the symbols $>$, $<$, $=$ (e.g., $0.83 > 0.8$ or $0.19 < 0.2$).
 - Order the value of decimals, from least to greatest and greatest to least (e.g., 0.83, 0.821, 0.8).

- Decimal numbers are another way of writing fractions (halves, fourths, fifths, and tenths). The Base-10 models concretely relate fractions to decimals (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, decimal circles money).
- Provide a fraction model (halves, fourths, fifths, and tenths) and ask students for its decimal equivalent.

Provide a decimal model and ask students for its fraction equivalent (halves, fourths, fifths, and tenths).

Essential New Vocabulary

- decimals
- decimal point
- decimal number
- tenths
- hundredths
- thousandths
- (given a) model

Released Test Items

What is 13.73 rounded to the nearest tenth?

- A** 13.0
- B** 13.7
- C** 13.8
- D** 14.0

<p>Standard</p>	<p>MATH 4.4</p> <p>The student will</p> <p>a) estimate sums, differences, products, and quotients of whole numbers. b) add, subtract, and multiply whole numbers. c) divide whole numbers, finding quotients with and without remainders. d) solve single-step and multistep addition, subtraction, and multiplication problems with whole numbers.</p> <p>Timeframe: Q 1 (8 days) Q 2 (23 days)</p>	<p>Essential Understandings</p> <p>All students should</p> <ul style="list-style-type: none"> • Develop and use strategies to estimate whole number sums and differences and to judge the reasonableness of such results. • Understand that addition and subtraction are inverse operations. • Understand that division is the operation of making equal groups or equal shares. When the original amount and the number of shares are known, divide to find the size of each share. When the original amount and the size of each share are known, divide to find the number of shares. • Understand that multiplication and division are inverse operations. • Understand various representations of division and the terms used in division are <i>dividend</i>, <i>divisor</i>, and <i>quotient</i>. <p style="text-align: center;"><i>dividend ÷ divisor = quotient</i></p> $\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array}$ <ul style="list-style-type: none"> • Understand how to solve single-step and multistep problems using whole number operations.
<p>Strand</p>	<p>Computation and Estimation</p>	
<p>Essential Knowledge and Skills</p>	<p>a) Estimate whole number sums, differences, products, and quotients.</p> <p>b) Refine estimates by adjusting the final amount, using terms such as <i>closer to</i>, <i>between</i>, and <i>a little more than</i>.</p> <p>c) Determine the sum or difference of two whole numbers, each 999,999 or less, in vertical and horizontal form with or without regrouping, using paper and pencil, and using a calculator.</p> <p>d) Estimate and find the products of two whole numbers when one factor has two digits or fewer and the other factor has three digits or fewer, using paper and pencil and calculators.</p> <p>e) Estimate and find the quotient of two whole numbers, given a one-digit divisor and a two- or three-digit dividend.</p> <p>f) Solve single-step and multistep problems using whole number operations.</p> <p>g) Verify the reasonableness of sums, differences, products, and quotients of whole numbers using estimation.</p>	<p>Teacher Notes</p> <p>See next page for notes.</p>

	Resource Materials	Released Test Items	Individual Teacher Notes	
Resources	<p><u>Envision Textbook</u></p> <p>4.4a</p> <ul style="list-style-type: none"> • SE: 32-33, 39, 50 (Set B), 100-103, 144, 145, 166, 167 • SEVH: VA 19 <p>4.4b</p> <ul style="list-style-type: none"> • SE: 26-31, 36-46, 53-69, 94-99, 106-118, 140-143, 146-157 • SEVH: VA 18 <p>4.4c</p> <ul style="list-style-type: none"> • SE: 76-79, 80-85, 86-88, 164, 165, 168-173, 174-181, 186, 187 • SEVH: VA 20 <p>4.4d</p> <ul style="list-style-type: none"> • SE: 20, 21, 34, 35, 102-104, 116-118, 134, 135, 156, 157, 186, 187 • SEVH: VA 22 • http://ttaonline.org/ 	<p>2010: 1, 2, 5, 6, 8, 10, 12</p> <p>2009: 1, 2, 4, 6, 7, 8, 12</p> <p>2008: 1, 7, 8, 9, 10, 11, 12</p> <p>2007: 1, 2, 3, 5, 6, 7, 9</p>		

Teacher Notes

- A sum is the result of adding two or more numbers.
- A difference is the amount that remains after one quantity is subtracted from another.
- An estimate is a number close to an exact solution. An estimate tells about how much or about how many.
- Different strategies for estimating include using compatible numbers to estimate sums and differences and using front-end estimation for sums and differences.
 - Compatible numbers are numbers that are easy to work with mentally. Number pairs that are easy to add or subtract are compatible. When estimating a sum, replace actual numbers with compatible numbers (e.g., $52 + 74$ can be estimated by using the compatible numbers $50 + 75$). When estimating a difference, use numbers that are close to the original numbers. Tens and hundreds are easy to subtract (e.g., $83 - 38$ is close to $80 - 40$).
 - The front-end strategy for estimating is computing with the front digits. Front-end estimation for addition can be used even when the addends have a different number of digits. The procedure requires the addition of the values of the digits in the greatest of the smallest number. For example:

$$\begin{array}{r} 2367 \rightarrow 2300 \\ 243 \rightarrow 200 \\ + 1186 \rightarrow + 1100 \\ \hline 3600 \end{array}$$

- Front-end or leading-digit estimation always gives a sum less than the actual sum; however, the estimate can be adjusted or refined so that it is closer to the actual sum.
 - Addition is the combining of quantities; it uses the following terms:
 - Compatible numbers: replace factors with compatible numbers, and then multiply. Opportunities for students to discover patterns with 10 and powers of 10 should be provided.
- $$\begin{array}{r} 64 \rightarrow 64 \\ \times 11 \rightarrow \times 10 \end{array}$$
- Division is the operation of making equal groups or equal shares. When the original amount and the number of shares are known, divide to find the size of each share. When the original amount and the size of each share are known, divide to find the number of shares. Both situations may be modeled with Base-10 manipulatives.
 - Multiplication and division are inverse operations.

- Terms used in division are *dividend*, *divisor*, and *quotient*.

$$\text{dividend} \div \text{divisor} = \text{quotient}$$

$$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array}$$

- Opportunities to invent division algorithms help students make sense of the algorithm. Teachers should teach division by various methods such as repeated multiplication and subtraction (partial quotients) before teaching the traditional long division algorithm.

$$\begin{array}{r} \text{addend} \rightarrow 45,623 \\ \text{addend} \rightarrow + 37,846 \\ \hline \text{sum} \rightarrow 83,469 \end{array}$$

- Subtraction is the inverse of addition; it yields the difference between two numbers and uses the following terms:

$$\begin{array}{r} \text{minuend} \rightarrow 45,698 \\ \text{subtrahend} \rightarrow - 32,741 \\ \hline \text{difference} \rightarrow 12,957 \end{array}$$

- Before adding or subtracting with paper and pencil, addition and subtraction problems in horizontal form should be rewritten in vertical form by lining up the places vertically.
- Using Base-10 materials to model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from the concrete to the pictorial, to the abstract. Regrouping is used in addition and subtraction algorithms. In addition, when the sum in a place is 10 or more, is used to regroup the sums so that there is only one digit in each place. In subtraction, when the number (minuend) in a place is not enough from which to subtract, regrouping is required.
- A certain amount of practice is necessary to develop fluency with computational strategies for multidigit numbers; however, the practice must be meaningful, motivating, and systematic if students are to develop fluency in computation, whether mentally, with manipulative materials, or with paper and pencil.
- Calculators are an appropriate tool for computing sums and differences of large numbers, particularly when mastery of the algorithm has been demonstrated.
- The terms associated with multiplication are

$$\begin{array}{r} \text{factor} \rightarrow 376 \\ \text{factor} \rightarrow \times 23 \\ \hline \text{product} \rightarrow 8,648 \end{array}$$
- One model of multiplication is repeated addition.

- Another model of multiplication is the “Partial Product” model.

$$\begin{array}{r}
 24 \\
 \times 3 \\
 \hline
 12 \leftarrow \text{Multiply the ones: } 3 \times 4 = 12 \\
 + 60 \leftarrow \text{Multiply the tens: } 3 \times 20 = 60 \\
 \hline
 72
 \end{array}$$

- Another model of multiplication is the “Area Model” (which also represents partial products) and should be modeled first with Base-10 blocks. (e.g., 23 x 68)
- Students should continue to develop fluency with single-digit multiplication facts and their related division facts.
- Calculators should be used to solve problems that require tedious calculations.
- Estimation should be used to check the reasonableness of the product. Examples of estimation strategies include the following:
 - The front-end method: multiply the front digits and then complete the product by recording the number of zeros found in the factors. It is important to develop understanding of this process before using the step-by-step procedure.

$$\begin{array}{r}
 523 \rightarrow 500 \\
 \times 31 \rightarrow \times 30 \\
 \hline
 15,000
 \end{array}$$

- This is $3 \times 5 = 15$ with 3 zeros.

Essential New Vocabulary

- | | | |
|-----------------|----------------------|----------------------------|
| • sums | • multistep problems | • front-end |
| • difference(s) | • closer to | • leading-digit estimation |
| • estimate | • between | • factor |
| • solution | • a little more than | • product |
| • dividend | • product(s) | • inverse operations |
| • divisor | • quotients of | • between |
| • quotient | • remainders | • regroup |

Released Test Items

741, 409 – 23, 611 =

A 717,798

B 718,798

C 722,898

D 727,898

<p>Standard</p>	<p>MATH 4.5</p> <p>The student will</p> <ul style="list-style-type: none"> a) determine common multiples and factors, including least common multiple and greatest common factor. b) add and subtract fractions having like and unlike denominators that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors. c) add and subtract with decimals. d) solve single-step and multistep practical problems involving addition and subtraction with fractions and with decimals. <p>Timeframe: Q 2 (5 days) Q 3 (12 days)</p>	<p>Essential Understandings</p> <p>All students should</p> <ul style="list-style-type: none"> • Understand and use common multiples and common factors for simplifying fractions. • Develop and use strategies to estimate addition and subtraction involving fractions and decimals. • Use visual models to add and subtract with fractions and decimals.
<p>Strand</p>	<p>Computation and Estimation</p>	<p>Teacher Notes</p>
<p>Essential Knowledge and Skills</p>	<ul style="list-style-type: none"> a) Find common multiples and common factors of numbers. b) Determine the least common multiple and greatest common factor of numbers. c) Use least common multiple and/or greatest common factor to find a common denominator for fractions. d) Add and subtract with fractions having like denominators whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors. e) Add and subtract with fractions having unlike denominators whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors. f) Solve problems that involve adding and subtracting with fractions having like and unlike denominators whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors. g) Add and subtract with decimals through thousandths, using concrete materials, pictorial representations, and paper and pencil. h) Solve single-step and multistep problems that involve adding and subtracting with fractions and decimals through thousandths. 	<p>See next page for notes.</p>

	Resource Materials	Released Test Items	Individual Teacher Notes	
Resources	<p><u>Envision Textbook</u></p> <p>4.5a</p> <ul style="list-style-type: none"> • SE: 58, 59, 182, 183, 228, 229 • SEVH: VA 6-VA 7, VA 21 <p>4.5b</p> <ul style="list-style-type: none"> • SE: 250-257 • SEVH: VA 30, VA 46 <p>4.5c</p> <ul style="list-style-type: none"> • SE: 296-298, 300-303 <p>4.5d</p> <ul style="list-style-type: none"> • SE: 238-240, 258-261, 293 • SEVH: VA12 and 13, VA 47, VA 50 • http://ttaconline.org/ 	<p>2010: 3, 4, 7, 9, 11</p> <p>2009: 3, 5, 9, 10, 11</p> <p>2008: 3, 4, 5, 6</p> <p>2007: 4, 8, 10, 11, 12</p>		

Teacher Notes

- A factor of a number is an integer that divides evenly into that number with a remainder of zero.
- A factor of a number is a divisor of the number.
- A multiple of a number is the product of the number and any natural number.
- A common factor of two or more numbers is a divisor that all of the numbers share.
- The least common multiple of two or more numbers is the smallest common multiple of the given numbers.
- The greatest common factor of two or more numbers is the largest of the common factors that all of the numbers share.
- Students should investigate addition and subtraction with fractions, using a variety of models (e.g., fraction circles, fraction strips, rulers, linking cubes, pattern blocks).
- When adding or subtracting with fractions having like denominators, add or subtract the numerators and use the same denominator. Write the answer in simplest form using common multiples and factors.
- When adding or subtracting with fractions having unlike denominators, rewrite them as fractions with a common denominator. The least common multiple (LCM) of the unlike denominators is a common denominator (LCD). Write the answer in simplest form using common multiples and factors.
- Addition and subtraction of decimals may be explored, using a variety of models (e.g., 10-by-10 grids, number lines, money).
- For decimal computation, the same ideas developed for whole number computation may be used, and these ideas may be applied to decimals, giving careful attention to the placement of the decimal point in the solution. Lining up tenths to tenths, hundredths to hundredths, etc. helps to establish the correct placement of the decimal.
- Fractions may be related to decimals by using models (e.g., 10-by-10 grids, decimal squares, money).

Essential New Vocabulary

- multiple
- common factor
- least common multiple
- greatest common factor
- common multiples
- integer
- like denominator
- unlike denominator

Released Test Items

Alyssa watched 3.5 hours of television last week. This week, she watched 4.7 hours of television. How many more hours did Alyssa watch television this week than last week?

- A** 0.2
- B** 0.8
- C** 1.2
- D** 8.2

<p>Standard</p>	<p>MATH 4.6</p> <p>The student will</p> <p>a) estimate and measure weight/mass and describe the results in U.S. Customary and metric units as appropriate.</p> <p>b) identify equivalent measurements between units within the U.S. Customary system (ounces, pounds, and tons) and between units within the metric system (grams and kilograms).</p> <p>Timeframe: Q 3 (4 days)</p>			<p>Essential Understandings</p> <p>All students should</p> <ul style="list-style-type: none"> • Use benchmarks to estimate and measure weight/mass. • Identify equivalent measures between units within the U.S. Customary and between units within the metric measurements. <p>Teacher Notes</p> <ul style="list-style-type: none"> • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object’s mass, although they use the term <i>weight</i> (e.g., “How much does it weigh?” versus “What is its mass?”). • Balances are appropriate measuring devices to measure weight in U.S. Customary units (ounces, pounds) and mass in metric units (grams, kilograms). • Practical experience measuring the mass of familiar objects helps to establish benchmarks and facilitates the student’s ability to estimate weight/mass. • Students should estimate the mass/weight of everyday objects (e.g., foods, pencils, book bags, shoes), using appropriate metric or U.S. Customary units. 						
<p>Strand</p>	<p>Measurement</p>									
<p>Essential Knowledge and Skills</p>	<p>a) Determine an appropriate unit of measure (e.g., ounce, pound, ton, gram, kilogram) to use when measuring everyday objects in both metric and U.S. Customary units.</p> <p>b) Measure objects in both metric and U.S. Customary units (e.g., ounce, pound, ton, gram, or kilogram) to the nearest appropriate measure, using a variety of measuring instruments.</p> <p>c) Record the mass of an object including the appropriate unit of measure (e.g., 24 grams).</p>									
<p>Resources</p>	<table border="1"> <thead> <tr> <th data-bbox="330 995 825 1076">Resource Materials</th> </tr> </thead> <tbody> <tr> <td data-bbox="330 1076 825 1463"> <p><u>Envision Textbook</u></p> <p>4.6a</p> <ul style="list-style-type: none"> • SE: 368, 369, 372, 378, 379, 396 (Set C) • SEVH:VA 42-43, VA 56 <p>4.6b</p> <ul style="list-style-type: none"> • SE: 368-369, 370-372, 378, 379, 380-382 </td> </tr> </tbody> </table>	Resource Materials	<p><u>Envision Textbook</u></p> <p>4.6a</p> <ul style="list-style-type: none"> • SE: 368, 369, 372, 378, 379, 396 (Set C) • SEVH:VA 42-43, VA 56 <p>4.6b</p> <ul style="list-style-type: none"> • SE: 368-369, 370-372, 378, 379, 380-382 	<table border="1"> <thead> <tr> <th data-bbox="825 995 1185 1076">Released Test Items</th> </tr> </thead> <tbody> <tr> <td data-bbox="825 1076 1185 1463"> <p>2010: 31</p> <p>2009: 22, 28</p> <p>2008: 31</p> <p>2007: 27, 31</p> </td> </tr> </tbody> </table>	Released Test Items	<p>2010: 31</p> <p>2009: 22, 28</p> <p>2008: 31</p> <p>2007: 27, 31</p>	<table border="1"> <thead> <tr> <th data-bbox="1185 995 1669 1076">Individual Teacher Notes</th> </tr> </thead> <tbody> <tr> <td data-bbox="1185 1076 1669 1463"></td> </tr> </tbody> </table>	Individual Teacher Notes		
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Released Test Items										
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Individual Teacher Notes										

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| | <ul style="list-style-type: none">• SEVH: VA 42-43, VA 57 | | | |
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| | <ul style="list-style-type: none">• http://ttaonline.org/ | | | |
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Essential New Vocabulary

- metric unit(s)
- metric
- US customary units
- gram
- kilogram
- mass
- weight
- ounce
- pound
- ton

Released Test Items

Anna bought 1 pound of butter. Which of the following is equivalent to 1 pound?

- A** 10 ounces
- B** 16 ounces
- C** 28 ounces
- D** 36 ounces

<p>Standard</p>	<p>MATH 4.7</p> <p>The student will</p> <p>a) estimate and measure length, and describe the result in both metric and U.S. Customary units.</p> <p>b) identify equivalent measurements between units within the U.S. Customary system (inches and feet; feet and yards; inches and yards; yards and miles) and between units within the metric system (millimeters and centimeters; centimeters and meters; and millimeters and meters).</p> <p>Timeframe: Q 3 (4 days)</p>	<p>Essential Understandings</p> <p>All students should</p> <ul style="list-style-type: none"> • Use benchmarks to estimate and measure length. • Understand how to convert units of length between the U.S. Customary and metric systems, using ballpark comparisons. • Understand the relationship between U.S. Customary units and the relationship between metric units.
<p>Strand</p>	<p>Measurement</p>	<p>Teacher Notes</p>
<p>Essential Knowledge and Skills</p>	<p>a) Determine an appropriate unit of measure (e.g., inch, foot, yard, mile, millimeter, centimeter, and meter) to use when measuring everyday objects in both metric and U.S. Customary units.</p> <p>b) Estimate the length of everyday objects (e.g., books, windows, tables) in both metric and U.S. Customary units of measure.</p> <p>c) Measure the length of objects in both metric and U.S. Customary units, measuring to the nearest inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), foot, yard, mile, millimeter, centimeter, or meter, and record the length including the appropriate unit of measure (e.g., 24 inches).</p> <p>d) Compare estimates of the length of objects with the actual measurement of the length of objects.</p> <p>e) Identify equivalent measures of length between units within the U.S. Customary measurements and between units within the metric measurements.</p>	<ul style="list-style-type: none"> • Length is the distance along a line or figure from one point to another. • U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include rulers, yardsticks, and tape measures. Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter ruler, meter stick, and tape measure. • Practical experience measuring the length of familiar objects helps to establish benchmarks and facilitates the student’s ability to estimate length. • Students should estimate the length of everyday objects (e.g., books, windows, tables) in both metric and U.S. Customary units of measure. • When measuring with U.S. Customary units, students should be able to measure to the nearest part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), inch, foot, or yard.

	Resource Materials	Released Test Items	Individual Teacher Notes
Resources	<p><u>Envision Textbook</u></p> <p>4.7a</p> <ul style="list-style-type: none"> • SE: 364, 365, 372, 374, 375, 396 (Set A), 397 (Set E) • SEVH: VA 54 <p>4.7b</p> <ul style="list-style-type: none"> • SE: 364, 365, 370-372, 374, 375, 380-382 • SEVH: VA 55 • http://ttaonline.org L 	<p>2010: 24, 29</p> <p>2009: 30</p> <p>2008: 22, 23, 25, 30</p> <p>2007: 21, 23</p>	

Essential New Vocabulary		
<ul style="list-style-type: none"> • metric unit(s) • metric • US customary units • millimeter(s) 	<ul style="list-style-type: none"> • centimeter • meter • kilometer • length 	<ul style="list-style-type: none"> • inch • feet • yard • mile

Released Test Items

The bicycle path at the city park is 1 mile long. Which of the following is closest to a distance of 1 mile?

- A** 1.5 kilometers
- B** 1.5 meters
- C** 1.5 millimeters
- D** 1.5 centimeters

<p>Standard</p>	<p>MATH 4.8</p> <p>The student will</p> <p>a) estimate and measure liquid volume and describe the results in U.S. Customary units.</p> <p>b) identify equivalent measurements between units within the U.S. Customary system (cups, pints, quarts, and gallons).</p> <p>Timeframe: Q 3 (4 days)</p>			<p>Essential Understandings</p> <p>All students should</p> <ul style="list-style-type: none"> Understanding the “counting on” strategy for determining elapsed time in hour and minute increments over a 12-hour period from a.m. to a.m. or p.m. to p.m.
<p>Strand</p>	<p>Measurement</p>			<p>Teacher Notes</p>
<p>Essential Knowledge and Skills</p>	<p>a) Determine an appropriate unit of measure (cups, pints, quarts, gallons) to use when measuring liquid volume in U.S. Customary units.</p> <p>b) Estimate the liquid volume of containers in U.S. Customary units of measure to the nearest cup, pint, quart, and gallon.</p> <p>c) Measure the liquid volume of everyday objects in U.S. Customary units, including cups, pints, quarts, and gallons, and record the volume including the appropriate unit of measure (e.g., 24 gallons).</p> <p>d) Identify equivalent measures of volume between units within the U.S. Customary system.</p>			<p>Elapsed time is the amount of time that has passed between two given times.</p> <ul style="list-style-type: none"> Elapsed time should be modeled and demonstrated using analog clocks and timelines. Elapsed time can be found by counting on from the beginning time to the finishing time. <ul style="list-style-type: none"> Count the number of whole hours between the beginning time and the finishing time. Count the remaining minutes. Add the hours and minutes.
<p>Resources</p>	<p>Resource Materials</p> <p><u>Envision Textbook</u></p> <p>4.8a</p> <ul style="list-style-type: none"> SE: 366, 367, 370-372, 396 (Set C) SEVH: VA 40-41, VA 52 <p>4.8b</p> <ul style="list-style-type: none"> SE: 366, 367, 370-373 SEVH: VA 40-41, VA 53 	<p>Released Test Items</p> <p>2010: None</p> <p>2009: 23</p> <p>2008: 29</p> <p>2007: None</p>	<p>Individual Teacher Notes</p>	<ul style="list-style-type: none"> For example, to find the elapsed time between 10:15 a.m. and 1:25 p.m., count 10 minutes; and then, add 3 hours to 10 minutes to find the total elapsed time of 3 hours and 10 minutes.

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| | <ul style="list-style-type: none">• http://ttaconline.org/ | | | |
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Essential New Vocabulary

- metric unit
- metric
- US customary
- Milliliter
- Liter
- liquid volume
- cups
- pints
- quarts
- gallons

Released Test Items

Mr. Zander bought 1 pint of chocolate milk. Which of the following is equivalent to 1 pint?

- A** $\frac{1}{2}$ cup
- B** 1 quart
- C** 2 cups
- D** $\frac{1}{2}$ gallon

Standard	MATH 4.9 The student will determine elapsed time in hours and minutes within a 12-hour period. Timeframe: Q 1 (5 days)			Essential Understandings All students should <ul style="list-style-type: none"> Understanding the “counting on” strategy for determining elapsed time in hour and minute increments over a 12-hour period from a.m. to a.m. or p.m. to p.m. Teacher Notes <ul style="list-style-type: none"> Elapsed time is the amount of time that has passed between two given times. Elapsed time should be modeled and demonstrated using analog clocks and timelines. Elapsed time can be found by counting on from the beginning time to the finishing time. <ul style="list-style-type: none"> Count the number of whole hours between the beginning time and the finishing time. Count the remaining minutes. Add the hours and minutes. For example, to find the elapsed time between 10:15 a.m. and 1:25, count 10 minutes; and the, add 3 hours to 10 minutes to find the total elapsed time of 3 hours and 10 minutes.
Strand	Measurement			
Essential Knowledge and Skills	a) Determine the elapsed time in hours and minutes within a 12-hour period (times can cross between a.m. and p.m.). b) Solve practical problems in relation to time that has elapsed.			
Resources	Resource Materials	Released Test Items	Individual Teacher Notes	
	<u>Envision Textbook</u> 4.9 <ul style="list-style-type: none"> SE: 386-388, 392, 393, 399 (Set K), 399 (Set M) SEVH: VA 58 http://ttaonline.org/ 	None See 2011 Released Items When Available		

Essential New Vocabulary

- elapsed time
- a.m.
- p.m.

<p>Standard</p>	<p>MATH 4.10</p> <p>The student will</p> <p>a) identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices.</p> <p>b) identify representations of lines that illustrate intersection, parallelism, and perpendicularity.</p> <p>Timeframe: Q 1 (3 days)</p>			<p>Essential Understandings</p> <p>All students should</p> <ul style="list-style-type: none"> • Understand that points, lines, line segments, rays, and angles, including endpoints and vertices are fundamental components of noncircular geometric figures. • Understand that the shortest distance between two points on a flat surface is a line segment. • Understand that lines in a plane either intersect or are parallel. Perpendicularity is a special case of intersection. • Identify practical situations that illustrate parallel, intersecting, and perpendicular lines. <p>Teacher Notes</p> <p>See next page for notes.</p>
<p>Strand</p>	<p>Geometry</p>			
<p>Essential Knowledge and Skills</p>	<p>a) Identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices.</p> <p>b) Understand that lines in a plane can intersect or are parallel. Perpendicularity is a special case of intersection.</p> <p>c) Identify practical situations that illustrate parallel, intersecting, and perpendicular lines.</p>			
<p>Resources</p>	<p>Resource Materials</p>	<p>Released Test Items</p>	<p>Individual Teacher Notes</p>	
	<p>Envision Textbook</p> <p>4.10a</p> <ul style="list-style-type: none"> • SE: 196-197, 198, 199, 346, 347, 348-349 • SEVH: VA 8-9, 24 <p>4.10b</p> <ul style="list-style-type: none"> • SE: 196, 197, 206, 207, 212 (Set A), 240, 325 	<p>2007: 26, 28, 30, 32</p> <p>2008: 21, 26, 27</p> <p>2009: 21, 26, 29, 31</p> <p>2010: 23, 28,</p>		

- SEVH: VA 23

<http://ttaconline.org/>

Teacher Notes

- A point is a location in space. It has no length, width, or height. A point is usually named with a capital letter.
- A line is a collection of points going on and on infinitely in both directions. It has no endpoints. When a line is drawn, at least two points on it can be marked and given capital letter names. Arrows must be drawn to show that the line goes on in both directions infinitely (e.g., \overleftrightarrow{AB} , read as “the line AB”).
- A line segment is part of a line. It has two endpoints and includes all the points between those endpoints. To name a line segment, name the endpoints (e.g., \overline{AB} , read as “the line segment AB”).
- A ray is part of a line. It has one endpoint and continues infinitely in one direction. To name a ray, say the name of its endpoint first and then say the name of one other point on the ray (e.g., \overrightarrow{AB} , read as “the ray AB”).
- Two rays that have the same endpoint form an angle. This endpoint is called the *vertex*. Angles are found wherever lines and line segments intersect. An angle can be named in three different ways by using
 - three letters to name, in this order, a point on one ray, the vertex, and a point on the other ray
 - one letter at the vertex
 - a number written inside the rays of the angle.
- Intersecting lines have one point in common.
- Perpendicular lines are special intersecting lines that form right angles where they intersect.
- Parallel lines are lines that lie in the same plane and do not intersect. Parallel lines are always the same distance apart and do not share any points.
- Students should explore intersection, parallelism, and perpendicularity in both two and three dimensions. For example, students should analyze the relationships between the edges of a cube. Which edges are parallel? Which are perpendicular? What plane contains the upper left edge and the lower right edge of the cube? Students can visualize this by using the classroom itself to notice the lines formed by the intersection of the ceiling and walls, of the floor and wall, and of two walls.

Essential New Vocabulary

- points
- lines
- line segments
- rays
- angles
- endpoints
- vertices
- intersection
- parallelism
- perpendicularity
- intersecting lines
- perpendicular lines
- right angles
- parallel lines

What do a ray and a line segment have in common?

- F** Both have two endpoints.
- G** Both are part of a line.
- H** Both go on and on infinitely.
- J** Both include only 10 points.

Standard	MATH 4.11 The student will a) investigate congruence of plane figures after geometric transformations, such as reflection, translation, and rotation, using mirrors, paper folding and tracing. b) recognize the images of figures resulting from geometric transformations, such as translation, reflection, and rotation. Timeframe: Q 1 (2 days)			Essential Understandings All students should <ul style="list-style-type: none"> • Understand the meaning of the term <i>congruent</i>. • Understand how to identify congruent figures. • Understand that the orientation of figures does not affect congruency or noncongruency.
Strand	Geometry			
Essential Knowledge and Skills	Recognize the congruence of plane figures resulting from geometric transformations such as translation, reflection, and rotation, using mirrors, paper folding and tracing.			Teacher Notes See next page for notes.
Resources	Resource Materials	Released Test Items	Individual Teacher Notes	
	Envision Textbook 4.11a <ul style="list-style-type: none"> • SE: 448-449, 450-455, 460, 461 • SEVH: VA 44-45, VA 62 4.11b <ul style="list-style-type: none"> • SE: 448-455, 460, 461 • SEVH: VA-44, 45, VA 63 http://ttaconline.org/	2010: 21, 30 2009: 27 2008: 24, 28 2007: 24		

Teacher Notes

- The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.
 - **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
 - **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
 - **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
 - **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusion are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)
- Congruent figures are figures having exactly the same size and shape. Opportunities for exploring figures that are congruent and/or noncongruent can best be accomplished by using physical models.
- A translation is a transformation in which an image is formed by moving every point on a figure the same distance in the same direction.
- A reflection is a transformation in which a figure is flipped over a line called the *line of reflection*. All corresponding points in the image and preimage are equidistant from the line of reflection.
- A rotation is a transformation in which an image is formed by turning its preimage about a point.
- The resulting figure of a translation, reflection, or rotation is congruent to the original figure.

Essential New Vocabulary

- plane figures
- geometric transformations
- reflection
- transformations

- reflection
- translations
- rotation
- congruence

- noncongruent
- line of reflection
- transformation
- congruent figures

Released Test Items

Aaron drew these clouds. Which cloud does not appear to be congruent to the other three?

- F** Cloud 1
- G** Cloud 2
- H** Cloud 3
- J** Cloud 4

*See 2007 Released Test Question For Diagram (#28)

Standard	MATH 4.12 The student will a) define polygon. b) identify polygons with 10 or fewer sides. Timeframe: Q 1 (3 days)			Essential Understandings All students should <ul style="list-style-type: none"> Identify polygons with 10 or fewer sides in everyday situations. Identify polygons with 10 or fewer sides in multiple orientations (rotations, reflections, and translations of the polygons).
Strand	Geometry			
Essential Knowledge and Skills	a) Define and identify properties of polygons with 10 or fewer sides. b) Identify polygons by name with 10 or fewer sides in multiple orientations (rotations, reflections, and translations of the polygons).			Teacher Notes <ul style="list-style-type: none"> A polygon is a closed plane geometric figure composed of at least three line segments that do not cross. None of the sides are curved. A triangle is a polygon with three angles and three sides. A quadrilateral is a polygon with four sides. A rectangle is a quadrilateral with four right angles. A square is a rectangle with four sides of equal length. A trapezoid is a quadrilateral with exactly one pair of parallel sides. A parallelogram is a quadrilateral with both pairs of opposite sides parallel. A rhombus is a quadrilateral with 4 congruent sides. A pentagon is a 5-sided polygon. A hexagon is a 6-sided polygon. A heptagon is a 7-sided polygon. An octagon is an 8-sided polygon. A nonagon is a 9-sided polygon. A decagon is a 10-sided polygon.
Resources	Resource Materials	Released Test Items	Individual Teacher Notes	
	Envision Textbook 4.12a <ul style="list-style-type: none"> SE: 202-209 SEVH: VA 25 4.12b <ul style="list-style-type: none"> SE: 202-209 SEVH: VA 26 http://ttaconline.org/	2010: 26 2009: 2008: 2007: 30		

Essential New Vocabulary

- square
- sides
- polygon
- closed
- triangle
- quadrilateral
- rectangle
- trapezoid
- rhombus
- parallelogram
- pentagon
- hexagon
- heptagon
- octagon
- nonagon
- decagon
- kite
- sides

Released Test Items

A square is both a —

- F** rectangle and a triangle.
- G** parallelogram and a rhombus.
- H** triangle and a quadrilateral.
- J** polygon and a circle.

<p>Standard</p>	<p>MATH 4.13</p> <p>The student will</p> <p>a) predict the likelihood of an outcome of a simple event; and b) represent probability as a number between 0 and 1, inclusive.</p> <p>Timeframe: Q 3 (7 days)</p>	<p>Essential Understandings</p> <p>All students should</p> <ul style="list-style-type: none"> • Understand and apply basic concepts of probability. • Describe events as likely or unlikely and discuss the degree of likelihood, using the terms <i>certain</i>, <i>likely</i>, <i>equally likely</i>, <i>unlikely</i>, and <i>impossible</i>. • Predict the likelihood of an outcome of a simple event and test the prediction. • Understand that the measure of the probability of an event can be represented by a number between 0 and 1, inclusive. <p>Teacher Notes</p> <p>See next page for notes.</p>
<p>Strand</p>	<p>Probability and Statistics</p>	
<p>Essential Knowledge and Skills</p>	<p>a) Model and determine all possible outcomes of a given simple event where there are no more than 24 possible outcomes, using a variety of manipulatives, such as coins, number cubes, and spinners.</p> <p>b) Write the probability of a given simple event as a fraction, where the total number of possible outcomes is 24 or fewer.</p> <p>c) Identify the likelihood of an event occurring and relate it to its fractional representation (e.g., impossible/0; equally likely/$\frac{1}{2}$; certain/1).</p> <p>d) Determine the outcome of an event that is least likely to occur (less than half) or most likely to occur (greater than half) when the number of possible outcomes is 24 or less.</p> <p>e) Represent probability as a point between 0 and 1, inclusively, on a number line.</p>	

	Resource Materials	Released Test Items	Individual Teacher Notes
Resources	<p><u>Envision Textbook</u></p> <p>4.13a</p> <ul style="list-style-type: none"> • SE: 468-474, 476, 477 • SEVH: VA 64 <p>4.13b</p> <ul style="list-style-type: none"> • SE: 472-474, 481 (Set C) • SEVH: VA 65 <p>http://ttaonline.org/</p>	<p>2010: 34, 36, 37, 39, 40</p> <p>2009: 33, 36, 37, 39</p> <p>2008: 33, 36, 37, 39, 40</p> <p>2007: 34, 35, 36, 39</p>	

Teacher Notes

- A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives.
- Probability is the chance of an event occurring.
- The probability of an event occurring is the ratio of desired outcomes to the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event = $\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$.
- The probability of an event occurring is represented by a ratio between 0 and 1. An event is “impossible” if it has a probability of 0 (e.g., the probability that the month of April will have 31 days). An event is “certain” if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning).
- When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time).
- Conduct experiments to determine the probability of an event occurring for a given number of trials (no more than 25 trials), using manipulatives (e.g., the number of times “heads” occurs when flipping a coin 10 times; the chance that when the names of 12 classmates are put in a shoebox, a name that begins with *D* will be drawn).
- Students should have opportunities to describe in informal terms (i.e., *impossible, unlikely, as likely as unlikely, equally likely, likely, and certain*) the degree of likelihood of an event occurring.
- Activities should include practical examples.

- For an event such as flipping a coin, the equally likely things that can happen are called *outcomes*. For example, there are two equally likely outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up.
- For another event such as spinning a spinner that is one-third red and two-thirds blue, the two outcomes, red and blue, are not equally likely. This is an unfair spinner (since it is not divided equally), therefore, the outcomes are not equally likely.

Essential New Vocabulary

- outcome
- simple event
- probability
- favorable outcomes
- possible outcomes
- impossible
- certain
- unlikely
- as likely
- as unlikely
- equally likely
- likely
- event
- likelihood (of an event)

Released Test Items

Terrell put 25 marbles in a bag. It is certain that the first marble taken from the bag will be red. Which is the number of red marbles in the bag?

- F** 0
- G** 5
- H** 20
- J** 25

<p>Standard</p>	<p>MATH 4.14</p> <p>The student will collect, organize, display, and interpret data from a variety of graphs.</p> <p>Timeframe: Q 2 (5 days)</p>	<p>Essential Understandings</p> <p>All students should</p> <ul style="list-style-type: none"> • Understand the difference between representing categorical data and representing numerical data. • Understand that line graphs show change over time (numerical data). • Understand that bar graphs should be used to compare counts of different categories (categorical data). • Understand how data displayed in bar and line graphs can be interpreted so that informed decisions can be made. • Understand that the title and labels of the graph provide the foundation for interpreting the data.
<p>Strand</p>	<p>Probability and Statistics</p>	
<p>Essential Knowledge and Skills</p>	<ul style="list-style-type: none"> a) Collect data, using, for example, observations, measurement, surveys, scientific experiments, polls, or questionnaires. b) Organize data into a chart or table. c) Construct and display data in bar graphs, labeling one axis with equal whole number increments of 1 or more (numerical data) (e.g., 2, 5, 10, or 100) and the other axis with categories related to the title of the graph (categorical data) (e.g., swimming, fishing, boating, and water skiing as the categories of "Favorite Summer Sports"). d) Construct and display data in line graphs, labeling the vertical axis with equal whole number increments of 1 or more and the horizontal axis with continuous data commonly related to time (e.g., hours, days, months, years, and age). Line graphs will have no more than 10 identified points along a continuum for continuous data. For example, growth charts showing age versus height place age on the horizontal axis (e.g., 1 month, 2 months, 3 months, and 4 months). e) Title or identify the title in a given graph and label or identify the axes. f) Interpret data from simple line and bar graphs by describing the characteristics of the data and the data as a whole (e.g., the category with the greatest/least, categories with the same number of responses, similarities and differences, the total number). Data points will be limited to 30 and categories to 8. g) Interpret the data to answer the question posed, and compare the answer to the prediction (e.g., "The summer sport preferred by most is swimming, which is what I predicted before collecting the data."). h) Write at least one sentence to describe the analysis and interpretation of the data, identifying parts of the data that have special characteristics, including categories with 	<p>Teacher Notes</p> <p>See next page for notes.</p>

	the greatest, the least, or the same.		
Resources	Resource Materials	Released Test Items	Individual Teacher Notes
	<u>Envision Textbook</u> 4.14 <ul style="list-style-type: none"> • SE: 402-407, 410, 411, 416, 417, 418-421 • SEVH: Va 59, VA 60 http://ttaconline.org/	2010: 33, 35, 38 2009: 34, 35, 38, 40 2008: 34, 35, 38 2007: 33, 37, 38, 40	

Teacher Notes

- Data analysis helps describe data, recognize patterns or trends, and make predictions.
- Investigations involving practical data should occur frequently, and data can be collected through brief class surveys or through more extended projects taking many days.
- Students formulate questions, predict answers to questions under investigation, collect and represent initial data, and consider whether the data answer the questions.
- Line graphs are used to show how two continuous variables are related. Line graphs may be used to show how one variable changes over time. If this one variable is not continuous, then a broken line is used. By looking at a line graph, it can be determined whether the variable is increasing, decreasing, or staying the same over time.
 - The values along the horizontal axis represent continuous data on a given variable, usually some measure of time (e.g., time in years, months, or days). The data presented on a line graph is referred to as “continuous data,” as it represents data collected over a continuous period of time.
 - The values along the vertical axis are the scale and represent the frequency with which those values occur in the data set. The values should represent equal increments of multiples of whole numbers, fractions, or decimals, depending upon the data being collected. The scale should extend one increment above the greatest recorded piece of data.
 - Each axis should be labeled, and the graph should be given a title.
 - A line graph tells whether something has increased, decreased, or stayed the same with the passage of time. Statements representing an analysis and interpretation of the characteristics of the data in the graph should be included (e.g., trends of increase and/or decrease, and least and greatest).
- Bar graphs should be used to compare counts of different categories (categorical data). Using grid paper ensures more accurate graphs.
 - A bar graph uses parallel, horizontal or vertical bars to represent counts for several categories. One bar is used for each category, with the length of the bar

- representing the count for that category.
- There is space before, between, and after the bars.
- The axis that displays the scale representing the count for the categories should extend one increment above the greatest recorded piece of data. Fourth-grade students should collect data that are recorded in increments of whole numbers, usually multiples of 1, 2, 5, 10, or 100.
- Each axis should be labeled, and the graph should be given a title.
- Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be written.

Essential New Vocabulary

- data analysis
- line graphs
- continuous
- not continuous
- horizontal axis
- vertical axis
- scale
- increments
- chart
- bar graphs
- title
- categories
- frequency

Released Test Items

The line graph shows how Joseph's height changed as he grew. Which of the following is closest to Joseph's height when he was 15 years old?

- F** 80 in.
- G** 70 in.
- H** 65 in.
- J** 60 in.

*See 2007 Released Test for Diagram (#38)

Standard	MATH 4.15 The student will recognize, create, and extend numerical and geometric patterns. Timeframe: Q 1 (2 days) Q 4 (5 days)			Essential Understandings All students should Understand that patterns and functions can be represented in many ways and described using words, tables, graphs, and symbols. Teacher Notes <ul style="list-style-type: none"> • Most patterning activities should involve some form of concrete materials to make up a pattern. • Students will identify and extend a wide variety of patterns, including rhythmic, geometric, graphic, numerical, and algebraic. The patterns will include both growing and repeating patterns. • Reproduction of a given pattern in a different representation, using symbols and objects, lays the foundation for writing the relationship symbolically or algebraically. • Tables of values should be analyzed for a pattern to determine the next value.
Strand	Patterns, Functions, and Algebra			
Essential Knowledge and Skills	a) Describe geometric and numerical patterns, using tables, symbols, or words. b) Create geometric and numerical patterns, using concrete materials, number lines, tables, and words. c) Extend geometric and numerical patterns, using concrete materials, number lines, tables, and words.			
Resources	Resource Materials	Released Test Items	Individual Teacher Notes	
	Envision Textbook 4.15 <ul style="list-style-type: none"> • SE: 96-97, 128-129, 130-135, 356-357 • SEVH: VA 38-39, VA 51 http://ttaconline.org/	2010: 42, 44, 46, 47, 49 2009: 41, 44, 46, 47, 48 2008: 42, 44, 46, 47, 50 2007: 41, 42, 44, 45		

Essential New Vocabulary

- geometric pattern
- numerical pattern

Released Test Items

Adrian is using a subtraction rule to make the number pattern shown below. If the pattern continues in the same way, what will be the 7th number in the pattern?

281 278 275 272 269

F 270

G 266

H 263

J 260

Standard	MATH 4.16 The student will a) recognize and demonstrate the meaning of equality in an equation: and b) investigate and describe the associative property for addition and multiplication. Timeframe: Q 4 (5 days)			Essential Understandings All students should <ul style="list-style-type: none"> Understand that mathematical relationships can be expressed using equations. Understand that quantities on both sides of an equation must be equal. Understand that the associative property for addition means you can change the groupings of three or more addends without changing the sum. Understand that the associative property for multiplication means you can change the groupings of three or more factors without changing the product. Teacher Notes <ul style="list-style-type: none"> Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers. The commutative property for addition states that changing the order of the addends does not affect the sum (e.g., $4 + 3 = 3 + 4$). Similarly, the commutative property for multiplication states that changing the order of the factors does not affect the product (e.g., $2 \times 3 = 3 \times 2$). The associate property for addition states that the sum stays the same when the grouping of addends is changed [e.g., $15 + (35 + 16) = (15 + 35) + 16$]. The associative property for multiplication states that the product stays the same when the grouping of factors is changed [e.g., $6 \times (3 \times 5) = (6 \times 3) \times 5$].
Strand	Patterns, Functions, and Algebra			
Essential Knowledge and Skills	a) Recognize and demonstrate that the equals sign (=) relates equivalent quantities in an equation. b) Write an equation to represent equivalent mathematical relationships (e.g., $4 \times 3 = 2 \times 6$). c) Recognize and demonstrate appropriate use of the equals sign in an equation. d) Investigate and describe the associative property for addition as $(6 + 2) + 3 = 6 + (2 + 3)$. e) Investigate and describe the associative property for multiplication as $(3 \times 2) \times 4 = 3 \times (2 \times 4)$.			
Resources	Resource Materials	Released Test Items	Individual Teacher Notes	
	Envision Textbook 4.16a <ul style="list-style-type: none"> SE: 303, 432-437 SEVH: VA 61 4.16b <ul style="list-style-type: none"> SE: 28-31, 79 SEVH: VA 4-5, VA 17 	2010: 41, 43, 45, 48, 50 2009: 42, 43, 45, 49, 50 2008: 38, 41, 43, 45, 48, 49 2007: 43, 46, 47, 48, 49, 50		

	http://ttaonline.org/			
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Essential New Vocabulary

- equality
- equation
- associative property for addition
- associative property for multiplication
- equal sign (=)
- expression

Released Test Items

Which goes in the blank to make the statement true?

$$3 + 5 = \underline{\quad}$$

- A 5 (divided by) 3
- B $5 - 3$
- C 5×3
- D $5 + 3$

Additional Math Online Resources

[New York State Released Tests](#) (pdf format)

[VA DOE Training/Technical Assistance](#) (SOL related searches, King George is region 3)

[Making Connections to Content: Essential Vocabulary](#) (Dan Mulligan)

[Illuminations](#) (excellent lesson plans, interactive tools, NCTM standards)

[National Library of Virtual Manipulatives](#)

[Mathematics Activity Types](#) (technology integration with math, The College of William & Mary)

[Open Ended Assessment in Math](#)

[APEC Math Assessment Database](#)

[Math Dictionary for Kids](#)

[SOL Released Tests in Various Formats](#)

[Technology Tools for the 21st Century Classroom](#) (KGES portaportal, math tools)